Research proposal

Title: Multispecies models, beyond the two body interaction

Scientific supervisor: Emanuele Mingione **SSD**:MAT/07
Duration: 12 months

Funding: PRIN2022 Boltzmann Machines beyond the "independent identically distributed" Paradigm: a Mathematical Physics Approach. Project code: 2022B5LF52, CUP J53D2300369 0006 from the Italian Ministero dell'Università e Ricerca. The PRIN project is in collaboration with Università di Modena.

Brief description of the project

The aim of the project is the description of mathematical physics aspects for a class of statistical mechanics models, called multispecies models. The main feature of these models is the presence of different and interacting communities of elementary agents. We aim to describe the phase space, including the critical behaviour of multispecies models with deterministic and random interactions. The research line includes also the analysis of the related inverse problems with applications in social and economic science.

Description of the project

In recent times an increasing number of connections among people , both physical (transport) and virtual (world wide web, social media), profoundly changed our daily experience. Also Artificial Intelligence technologies play the role of an active community in the real life. Therefore networks where different communities interact become suitable models for various phenomena.

The aim of this research project is to study the mathematical properties of statistical mechanics models composed by a large number of agents belonging to different communities called multispecies models [CF11,BC13,OO18]. In particular we look for a description of phase transitions and critical phenomena and inverse problem. The theoretical work will be driven having in mind applications in social science [CG07, CKO22, BC15,CV20].

This project comprises two main lines of research:

WP1. Free energy and phase diagram, fluctuations of the order parameter (1-7 months)

We plan to study multispecies models with deterministic, mean field, quadratic and cubic interaction . In particular the task is to determine the free energy, phase transitions and limit theorems for the order parameter in the spirit of [CF11, MJ21]. The next step will be to consider the disorder setting] and try to relax the convexity assumption [BC13].The idea to overcome this obstacle is to combine a variant of the interpolation method called adaptive interpolation [BM19] with the synchronization method [Pa15]. The objective is to find a representation of the limiting free energy in terms of an infinite dimensional variational problem. The phase diagram can be obtained studying the solution of the above variational problem by means of theoretical and numerical methods.

WP2. Inverse problem, TAP approach (7-12 months)

The Ising inverse problem is a fundamental question in data science and machine learning theory [NZB17] . The task is to recover the parameters of the Hamiltonian (2) from the observation of a collection of spin configurations sampled from the Boltzmann-Gibbs measure. In the 80's Thouless-Anderson-Palmer (TAP) introduced a way to characterize the limiting behavior of the local magnetization of a Sherrington-Kirkpatrick, obtaining also a refinement of the naive mean field approximation for the Ising inverse problem [MM09,OW01], nowadays called TAP correction. We plan to derive a multispecies analogous of the TAP equations and obtain their convergence properties by Gaussian conditioning method [B13] . Further we aim to study numerically the robustness of the resulting multispecies TAP corrections to the Ising inverse problem.

Plan of activities

For WP1 (Months 1-7) the role of the researcher will be the development and analysis multispecies mode using the tools from statistical mechanics of mean field models,combinatorial methods, interpolation methods and cavity methods.
Months 1-2: literature review, critical analysis of the methodologies. Months 3-7 Free energy and limits theorems for deterministic models , extension to disordered models

For WP2 (Months 8-12) the role of the researcher will be the development and analysis of the inverse problem disorders systems, in particular the TAP corrections to non convex spin glasses.

The activity of dissemination of results will be carried out by giving seminars in Italian and international universities and participating as a speaker to the major conferences of the field.

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